



NE 112 *Linear algebra for nanotechnology engineering*

1.6.2a *i* versus *j*

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Outline

- In this topic, we will
 - Discuss the use of i and j to represent the square root of negative one





Why j ?

- Mathematicians represented the complex unit as i
 - This is likely related to it being *imaginary*
- In electrical engineering,
 - i generally represents current
 - Complex numbers are critical to understanding alternating current
 - Consequently, engineers adopted j





Why not j ?

- Some mathematicians object to using j because it is not i
- However, consider this:
 - Every mathematician accepts a construct called the quaternions
 - Quaternions have three imaginary units:

$$i^2 = j^2 = k^2 = -1$$

- Every quaternion is a real linear combination of 1, i , j , k :
 - E.g., $q = 3.2 - 4.5i + 6.1j + 9.7k$





Quaternions

- We can define the sum of two quaternions
 - E.g., if $q = 3.2 - 4.5i + 6.1j + 9.7k$
 $r = 1.7 + 6.4i + 9.2j - 8.5k$
then $q + r = 4.9 + 1.9i + 15.3j + 1.2k$
- Multiplication is more difficult
 - Suppose we define $ij = k$
 $jk = i$
 $ki = j$
 - Thus, $ij = -j = ik$
 $jjk = -k = ji$
 $kki = -i = kj$





Quaternion are not commutative

- For quaternions to work, we lose commutativity
 - Quaternions are **not** a field: $qr \neq rq$ in general...
- Quaternions were discovered around the same time as vectors
 - There was a debate as to whether we should use vectors or quaternions
 - Hint: vectors won out—this is not a course in quaternions





Sub-fields of the quaternions

- So, the quaternions are not commutative
 - However, all quaternions of the form $\alpha + \beta i$ are a subset that happen to behave like a field
 - Similarly, all quaternions of the form $\alpha + \gamma j$ are also a subset that happen to behave like a field
- Between these two sub-fields,
for all intents and purposes, we can treat
 $\alpha + \beta i$ and $\alpha + \beta j$ as being equivalent
 - Consequently, you can just tell such a mathematician that
“The complex numbers with i and the complex numbers with j are just two isometrically isomorphic sub-fields of the quaternions.”





Application of quaternions

- Interestingly enough,
 - there are still applications of quaternions in graphics
 - Consider the imaginary quaternions:
$$\beta i + \gamma j + \delta k$$
 - Quaternion multiplication can be used to approximate rotations in 3-space when the coordinate (β, γ, δ) is represented by the imaginary quaternion $\beta i + \gamma j + \delta k$
 - Matrices—which we will see in this course—allow us to define rotations, but these are subject to Gimbal lock
 - Quaternions require less memory and less computation for the same result





Summary

- In this topic, you now
 - Understand mathematicians use i and engineers use j
 - Know that there is no difference between these two usages
 - Just stay consistent
 - Incidentally, many programming languages allow you to choose your imaginary unit
 - Know you can joke about isometries between sub-fields of the quaternions
 - Are aware that quaternions can never-the-less be used in graphics





References

- [1] https://en.wikipedia.org/wiki/Imaginary_unit
- [2] <https://en.wikipedia.org/wiki/Quaternion>
- [3] https://en.wikipedia.org/wiki/Complex_number#Notation



Acknowledgments

None so far.



Colophon

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